Can We Use Validated Simulation Model Instead of a Large Number of Physical Tests for Confidence-Based Reliability Assessment?

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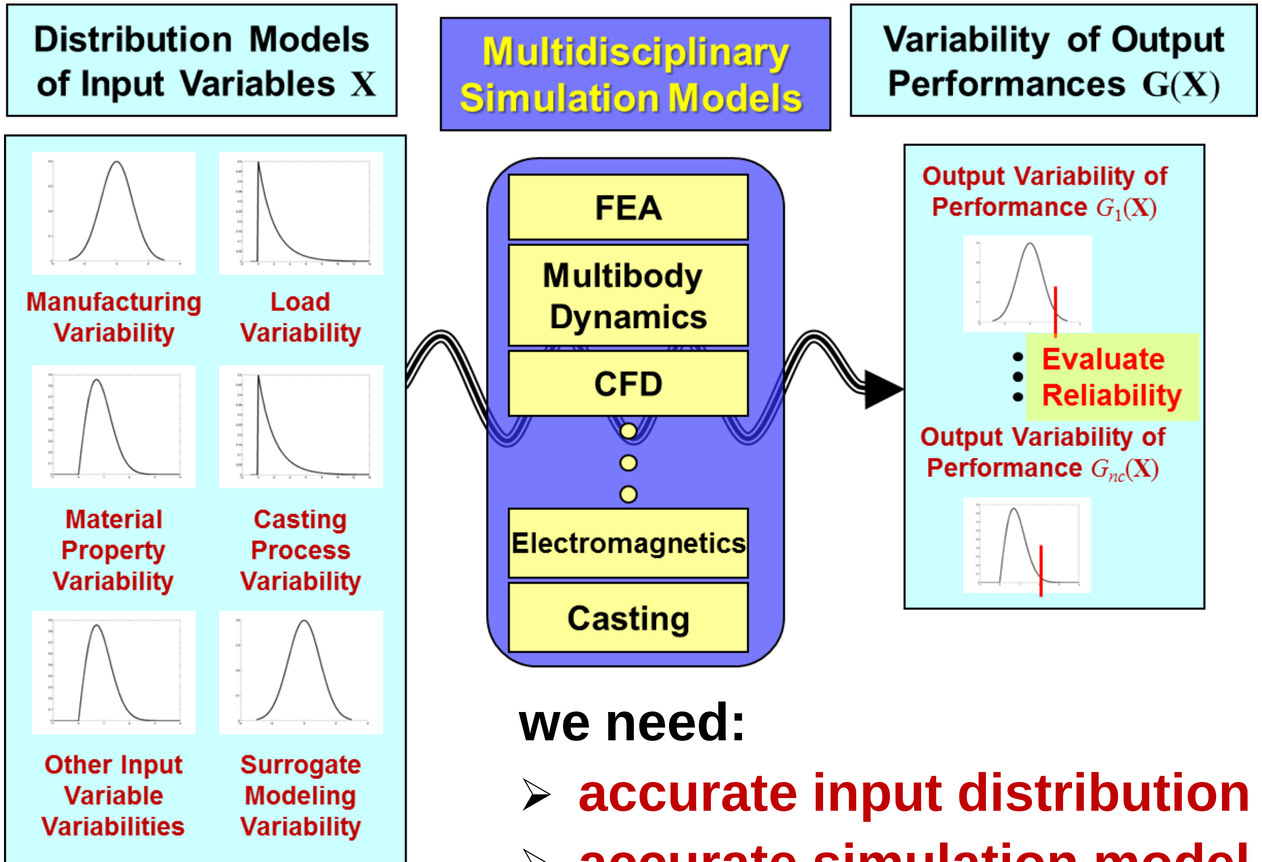
Contents

Motivation

- To evaluate product reliability accurately, a very large number of physical testing is required: **very expensive!**
- On the other hand, if we use simulation-based method,

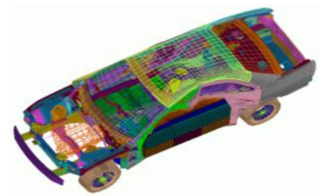


we need:

- **accurate input distribution models** &
- **accurate simulation model**.

Motivation (cont.)

- **In real practical applications,**
  - **We may have limited data for input distribution models.**
  - **Simulation model could be biased and may not depict the actual physical event.**



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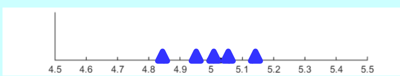
- **Thus, we may not have enough confidence if we use only simulation method for assessment of the reliability.**
- **Can we integrate the simulation method and a limited number of product output testing (for cost-effectiveness) for assessment of reliability with confidence?**
- **Insufficient input data ⇒ Induces uncertainty in input distribution models.**
- **Insufficient output test data ⇒ Can't generate proper target output distributions for validation of the simulation model.**

Challenge: Uncertainty in Input Distribution Model

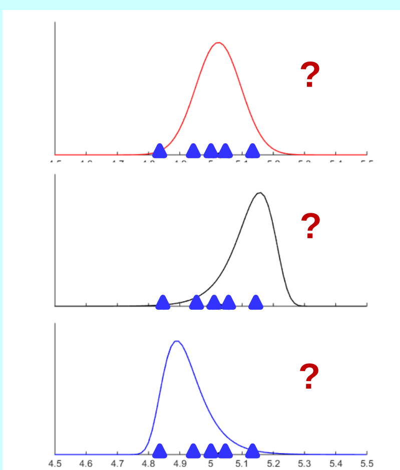
- Uncertainty in input distribution models propagate through (accurate) simulation model to yield **uncertainty in reliability.**

Distribution Models of Input Variables X

Insufficient Input Data



Which Input Distribution?

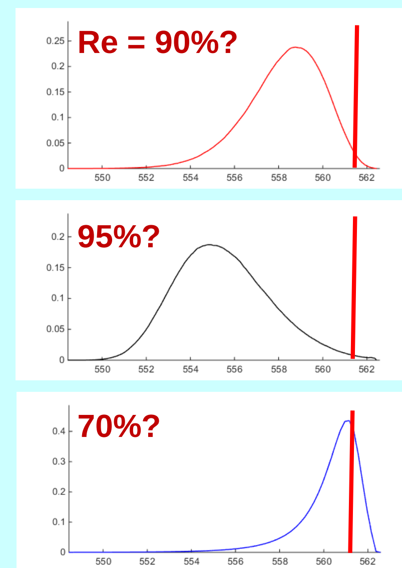


Multidisciplinary Simulation Models

- FEA
- Multibody Dynamics
- CFD
- Electromagnetics
- Casting

Variability of Output Performances G(X)

Output Distribution ?

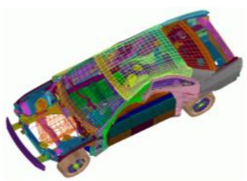


Consider possible *M* input distribution models & *M* corresponding reliabilities

Challenge: Biased Simulation Model & Output Uncertainty

- Model bias and uncertainty due to limited output test data

Biased Simulation



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Model Bias

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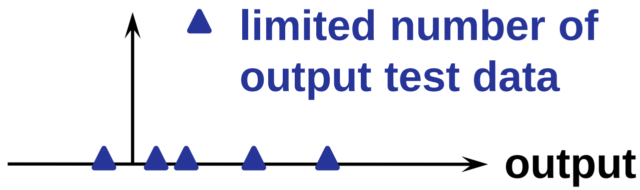
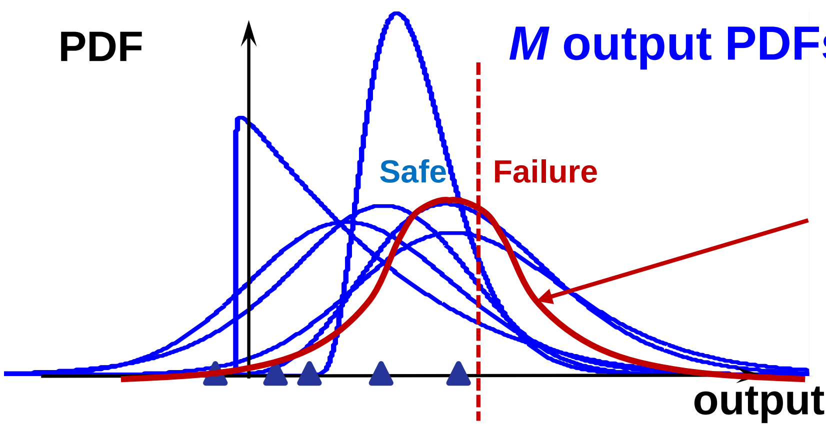
Uncertainty Due to Insufficient Output Test Data



Uncertain Output PDF and Reliability

M output PDFs & corresponding reliabilities



To find a confidence-based target output distribution against which the simulation output distribution can be validated.

Challenge: Combined Uncertainties

- Uncertainty induced by insufficient input and output data, and biased model should be combined.

Insufficient Input Data
+ **Bayesian Method**

Uncertainty in Input Distributions
- Input distribution parameters
- Input distribution types

+ Biased Simulation Model

Uncertainty in Biased Simulation Output Distribution

Insufficient Output Test Data

+ Hierarchical Bayesian Method

Uncertainty in Target Output Distribution

- Confidence-Based Reliability
- Confidence-Based Target Output Distribution

Obtain Validated Simulation Model

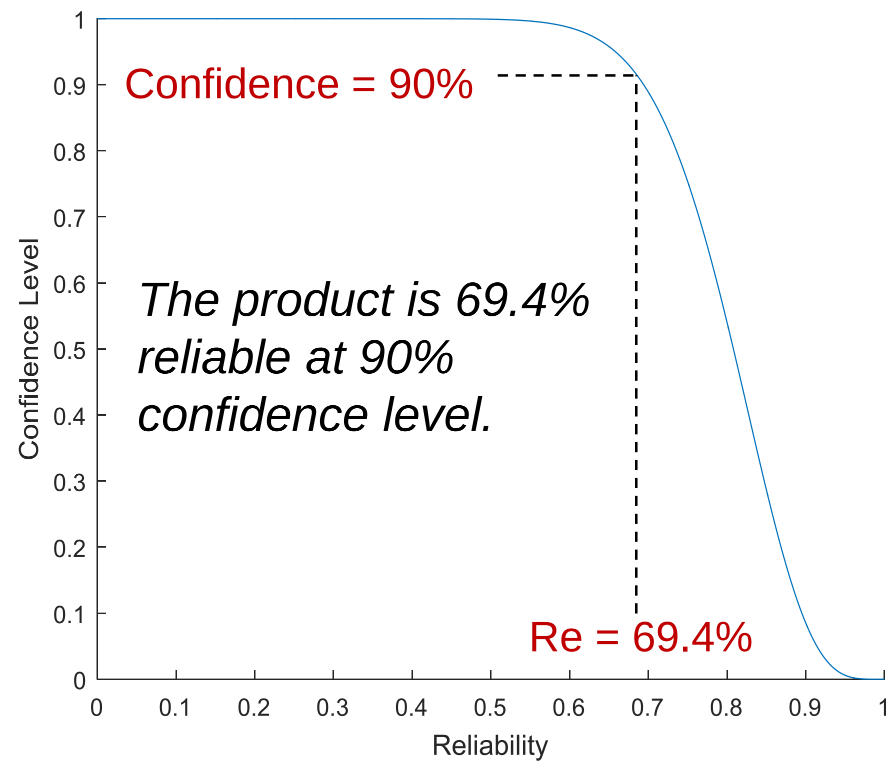
Quantification of Uncertain Input Distribution Models

Insufficient Input Data
+ Bayesian Method

Probability of
- Input distribution parameters
- Input distribution types

Simulation Model

Confidence-Based Reliability

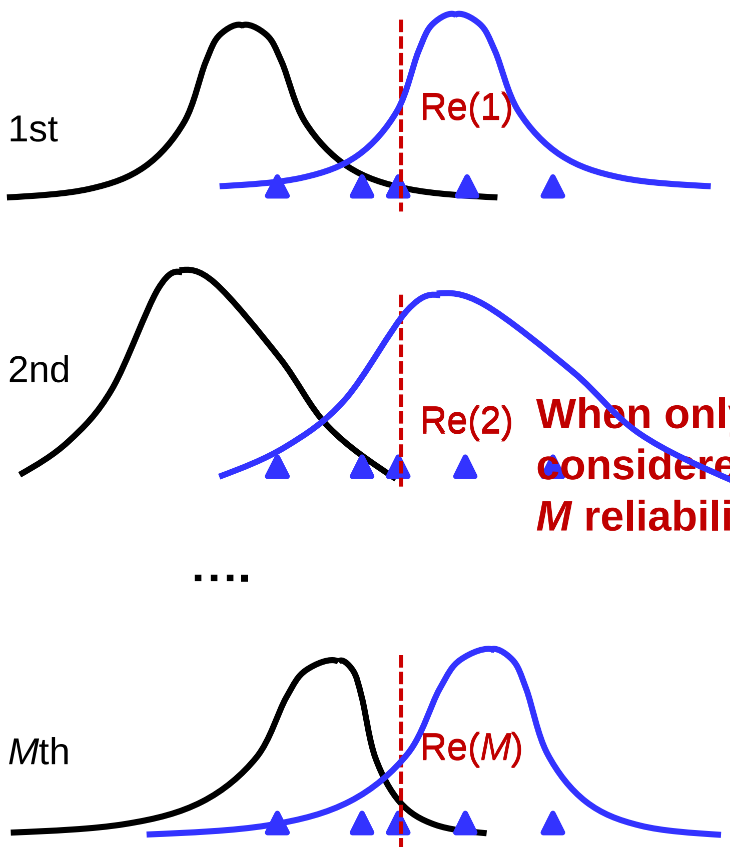


Uncertainty induced by insufficient input data is measured by the reliability at the target confidence level.

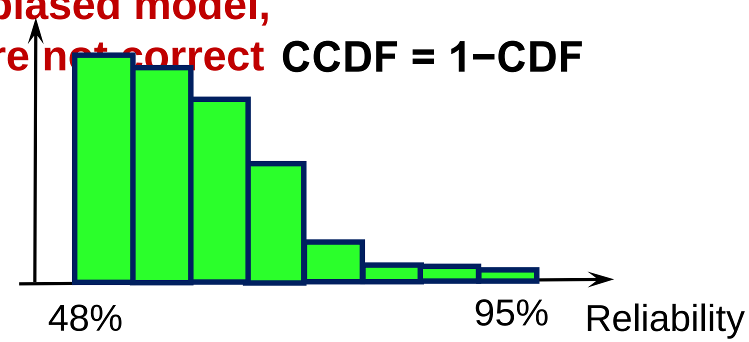
Combining Uncertainty Due to Limited Output Data with Uncertain Input Distribution Models

***M* possible biased simulation output PDFs due to limited input data** + **Limited output test data** → ***M* updated output PDFs and corresponding reliability values**

Complementary CDF (CCDF) of reliability (*i.e.*, uncertainty of variability) is obtained using Hierarchical Bayesian analysis considering both limited input and output test data.



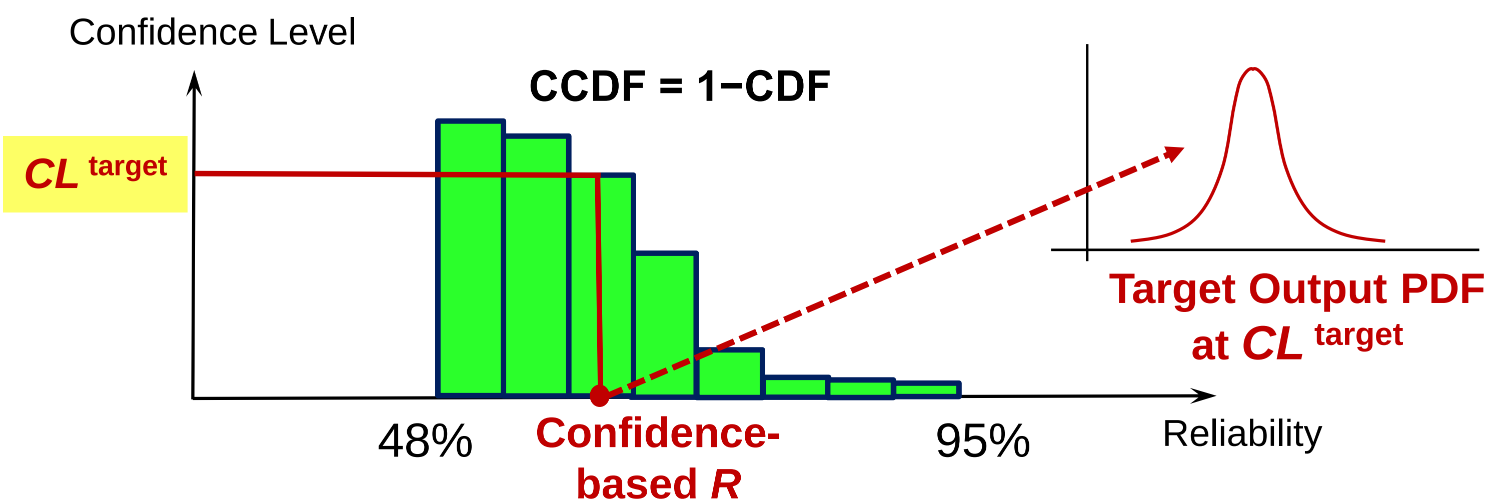
When only limited input data is considered with the biased model, *M* reliability values are not correct



Select Confidence-Based Reliability and Target Output PDF

Step 1) Specify a target confidence level that the user wants (*e.g.*, CL^{target} = 90%).

Step 2) The 90% percentile value from CCDF of reliability becomes confidence-based reliability (R). A higher percentile indicates a more conservative (*i.e.* lower) estimation of the reliability.

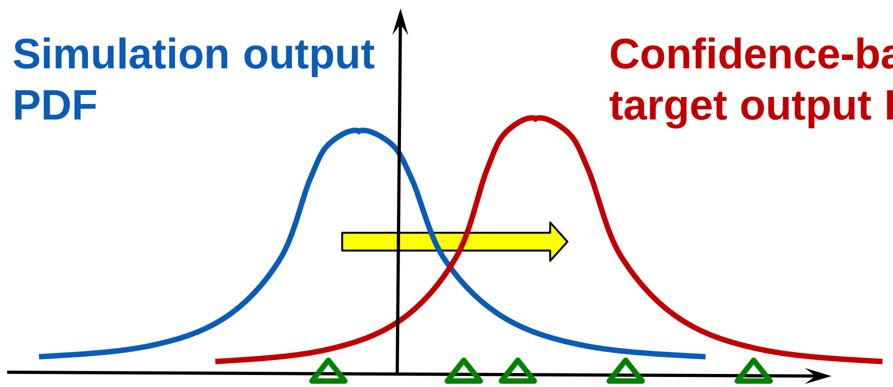


Step 3) Once confidence-based R is selected, the corresponding predicted output PDF becomes confidence-based target output PDF

Obtain Validated Simulation Model

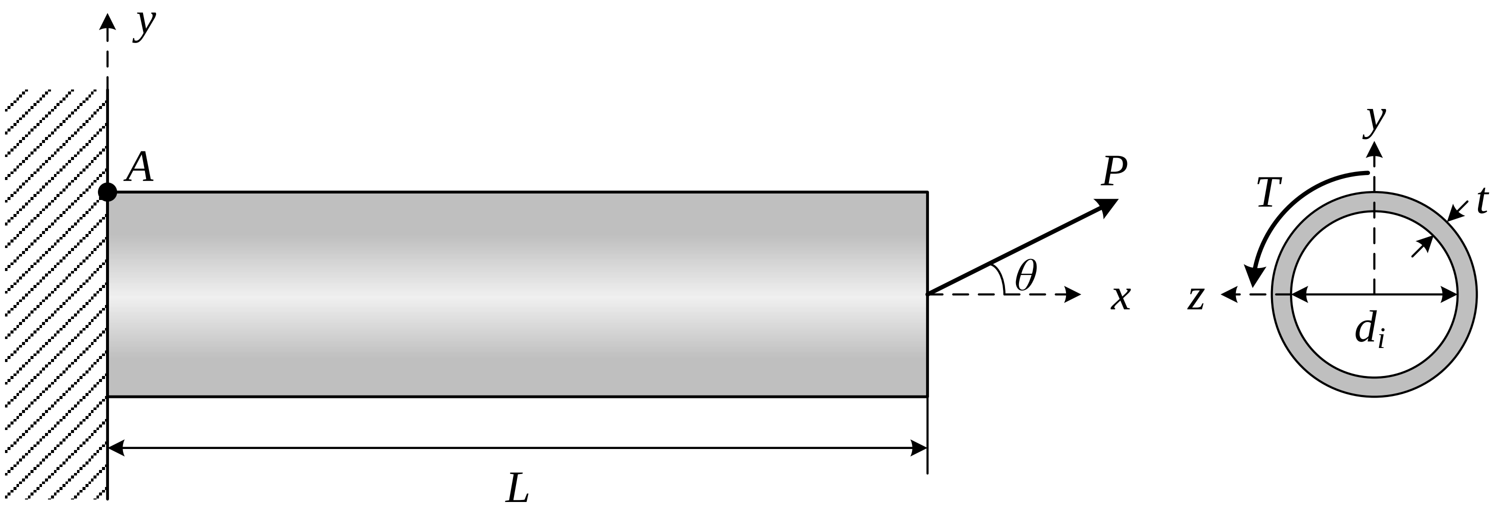
- **Model Validation Optimization**
  - Distance between simulation output PDF and confidence-based target output PDF is minimized for **model bias correction**
  - Obtain validated simulation model that satisfies confidence-based reliability at CL^{target}

Biasedness of Simulation Model & Validation Optimization



Example Description

❑ 9-D Cantilever Beam



- True constraint 1: Maximum stress at point ***A***
- True constraint 2: Torsional buckling
- Simulated (biased) constraint 1: True constraint 1 – **bias 1**
- Simulated (biased) constraint 2: True constraint 2 – **bias 2**

Example Description (cont.)

- 9-D Cantilever Beam

Input variable	Description	True distribution	Mean	Standard deviation	Known information
d_i	Inner diameter	Normal	189.8126 mm	10	We know true input distributions
t	Thickness		1.6611 mm	0.08	
L	Length		500 mm	10	
P	Force		50,000 N	5,000	Assume we do not know true input distributions
T	Torsion		10,000,000 N-mm	1,000,000	
θ	Angle		0°	10	
E	Young's modulus		200,000 MPa	10,000	
ν	Poisson's ration	Lognormal	0.26	0.026	
σ_Y	Yield strength	Normal	220 MPa	15.4	

Example Description (cont.)

❑ Mathematical Formulation of 9-D Cantilever Beam

- True constraint 1: Maximum stress at point A

$$G_1^{true} = \frac{\sigma_{max}}{S_y} - 1 = \frac{\sqrt{\sigma_x^2 + 3\tau_{xz}^2}}{S_y} - 1 \le 0 \quad where \quad \sigma_x = \frac{P\cos\theta}{A} + \frac{P\sin\theta L}{I}\frac{d_o}{2} \quad and \quad \tau_{xz} = \frac{Td_o}{4I}$$

$$\left(d_o = d_i + 2t,\ A = \pi\left(d_o - t\right)t\right)$$

- True constraint 2: Torsional buckling

$$G_2^{true} = 1 - \frac{T_{cr}}{T} \le 0 \le 0 \quad where \quad T_{cr} = \frac{\pi d_o^3 E}{3\left(1-\nu^2\right)^{0.75}}\left(\frac{t}{d_o}\right)^{2.5}$$

- Simulated (biased) constraint 1: True constraint 1 – bias 1

$$G_1 = g_1^{true} - B_1^{true} \quad where \quad B_1^{true} = \frac{\sqrt{0.01\sigma_x^2 + 0.036\tau_{xz}^2 + 0.37\left(P\cos\theta / \mathrm{A}\right)^2 + 5}}{S_y}$$

- Simulated (biased) constraint 2: True constraint 2 – bias 2

$$G_2 = g_2^{true} - B_2^{true} \quad where \quad B_2^{true} = \frac{\pi E}{T}\left(\frac{d_o^{0.5}}{6\left(1-\nu\right)^{0.7}} - \frac{1}{5\left(1-\nu^2\right)^{1.5}}\right)$$

Case Studies

Case 1: Limited Number of Data

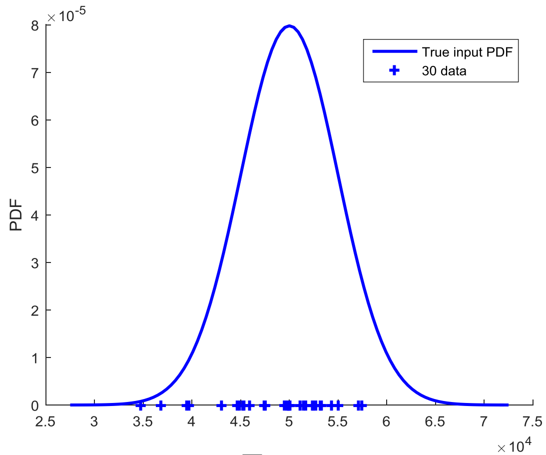
- **30 input test data for 6 variables**
- **20 output test data for each constraint**

Case 2: Smaller Number of Data

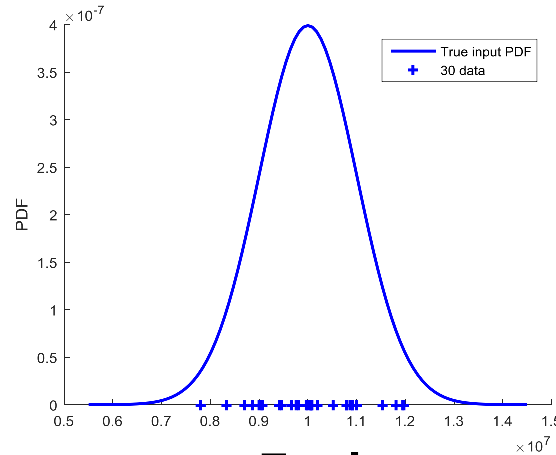
- **10 input test data for 6 variables**
- **5 output test data for each constraint**

Case 1: Limited Number of Input Data

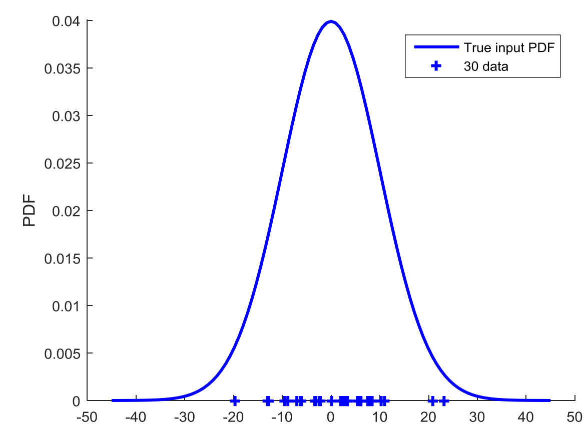
- **30 data for each variable** are randomly drawn from true input distributions – **to imitate carrying out testing**.



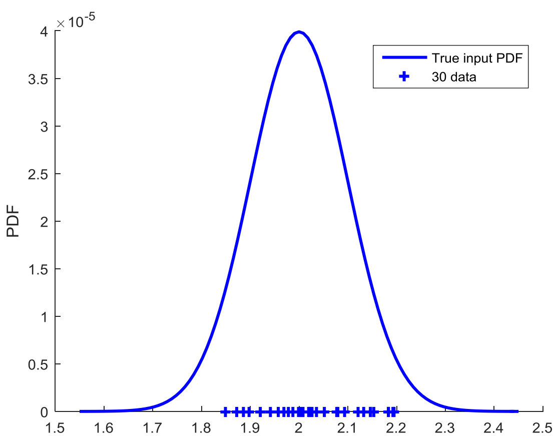
Force



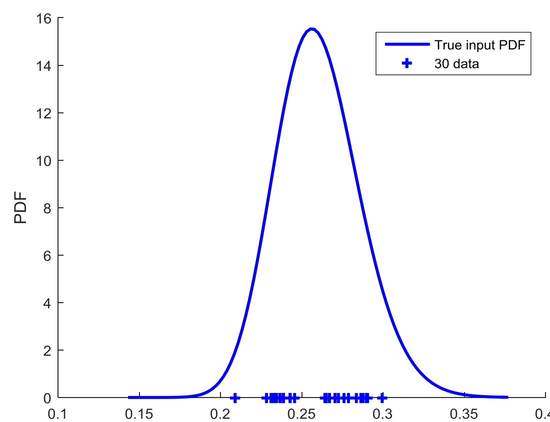
Torsion



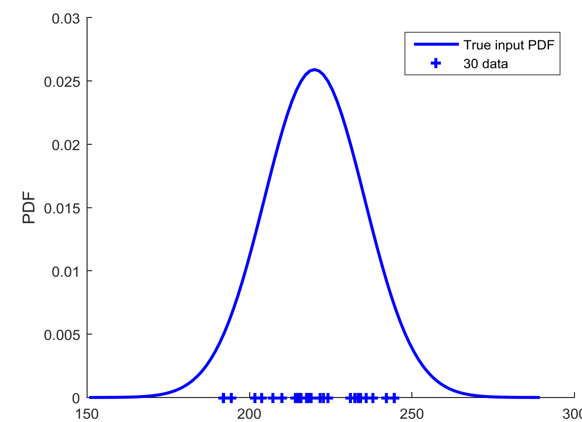
Angle



Young's modulus



Poisson's ratio



Yield strength

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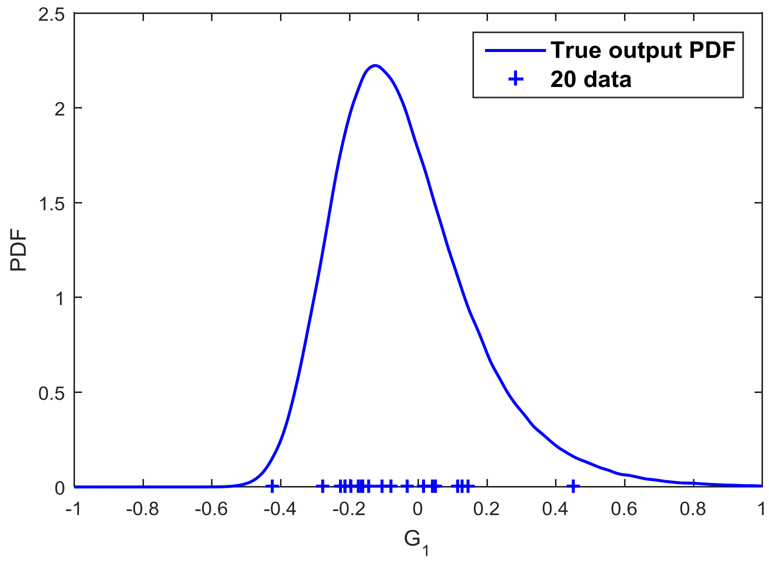
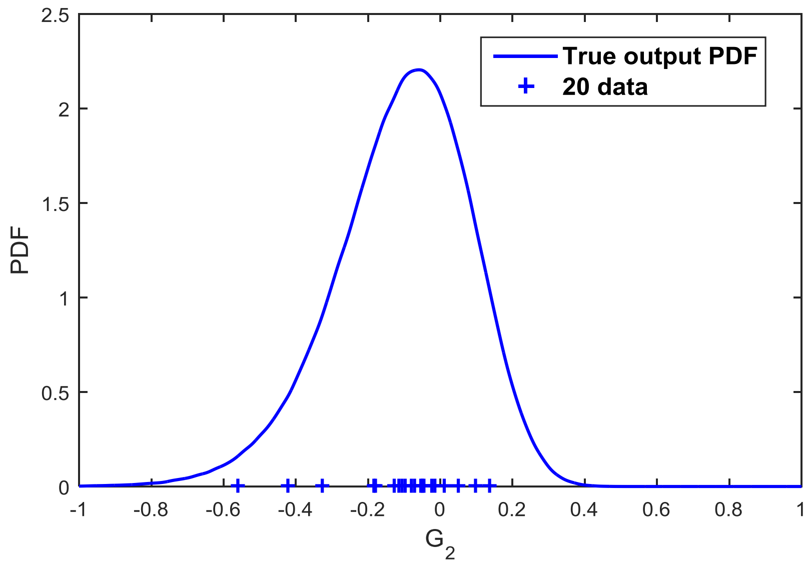
Case 1: Limited Number of Output Data

- **20 data for each constraint are randomly drawn from true output distributions – to imitate carrying out testing.**

Constraint 1

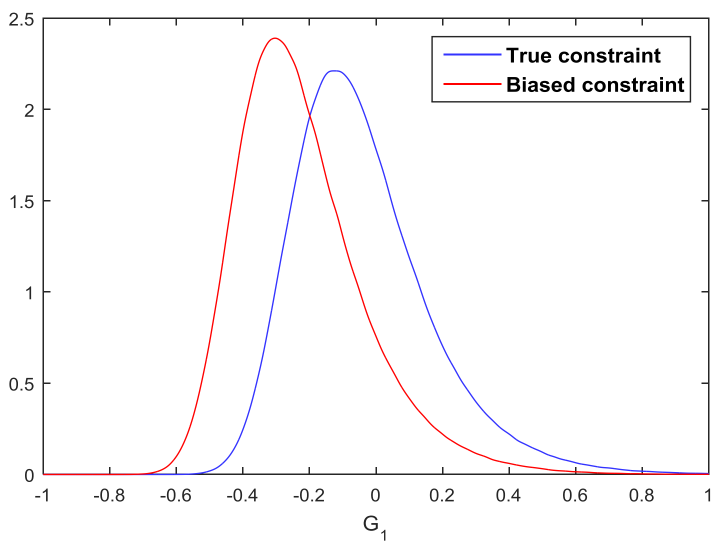


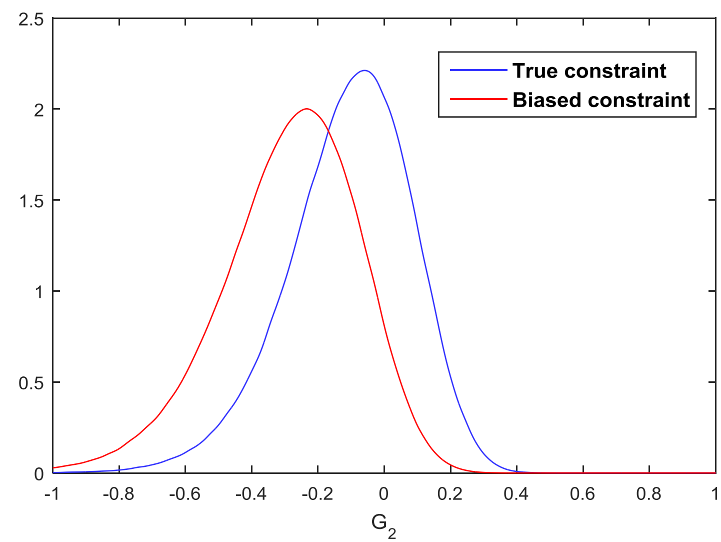
Constraint 2



Case 1: Reliability Using Biased Simulation Model

	True constraint (a)		Biased constraint (b)		Error ((a-b)/a×100)	
	$G_1 \leq 0$	$G_2 \leq 0$	$G_1 \leq 0$	$G_2 \leq 0$	$G_1 \leq 0$	$G_2 \leq 0$
Non-normalized output mean	210.14	10,958,273	167.31	12,615,870	**20.38%**	**−15.13%**
Reliability using true input dist.	**63.91%**	**69.97%**	-	-	-	
Reliability using best-fit input dist.	-	-	**88.86%**	**93.48%**	-	





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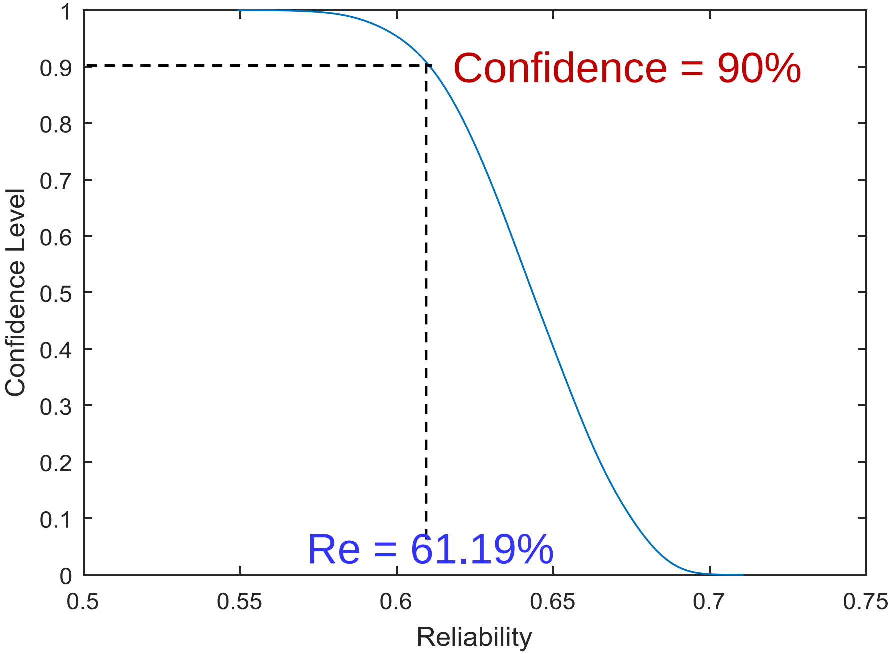
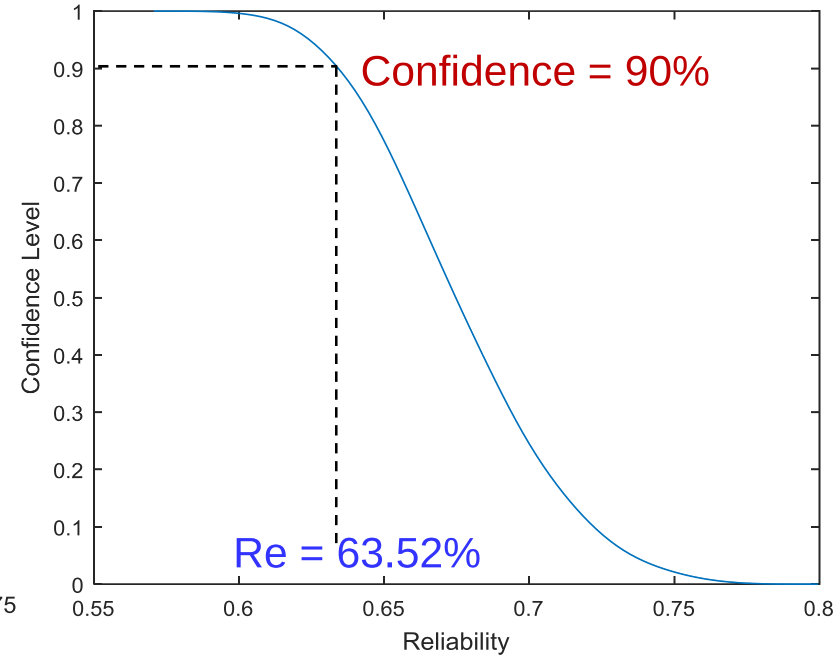
Case 1: Confidence-Based Reliability Assessment

Constraint 1



Constraint 2



RELIABILITY	Constraint 1	Constraint 2
Simulation Only Using Best-fit Input Model	88.86%	93.48%
Best-fit Using Output Test Data Only	67.36%	76.39%
Confidence-based (Simulation + Test Data)	61.19%	63.52%
True	63.91%	69.97%

Case 1: Confidence-Based Reliability Assessment

- **Repeated Test: 100 trials for numerical validation**
  - 30 data are randomly drawn from true input distribution
  - 20 data are randomly drawn from true output distribution
- **Target confidence level of 90% are satisfied**
- **Output best-fit method provides confidence level only 65%**

Method	**Confidence-based**		**Output Best-fit**	
Constraint	**1**	**2**	**1**	**2**
Mean	56.76%	62.31%	61.59%	67.34%
STD	5.65%	4.79%	8.46%	7.71%
Maximum	70.74%	78.54%	84.31%	92.01%
Minimum	45.52%	50.93%	44.37%	51.25%
True reliability	63.91%	69.97%	63.91%	69.97%
Successful trials	89	95	65	67

Each trial for Case 1 takes ~66 hours using 30 cores of Excalibur DSP (Army ARL).

Case Studies

Case 1: Limited Number of Data

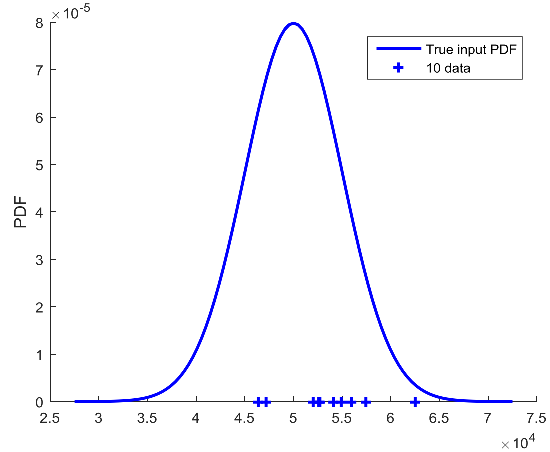
- **30 input test data for 6 variables**
- **20 output test data for each constraint**

Case 2: Smaller Number of Data

- **10 input test data for 6 variables**
- **5 output test data for each constraint**

Case 2: Smaller Number of Input Data

- **10 data for each variable are randomly drawn from true input distributions – to imitate carrying out testing.**



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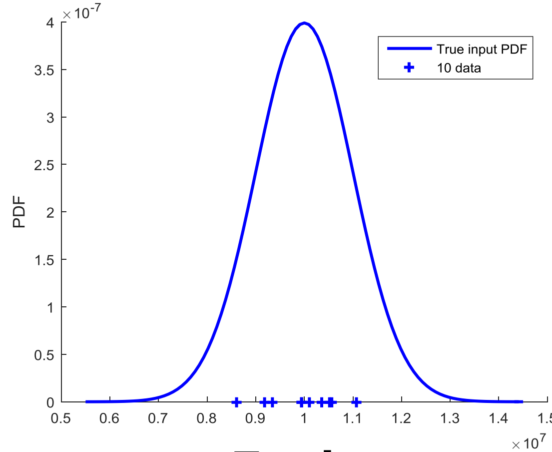
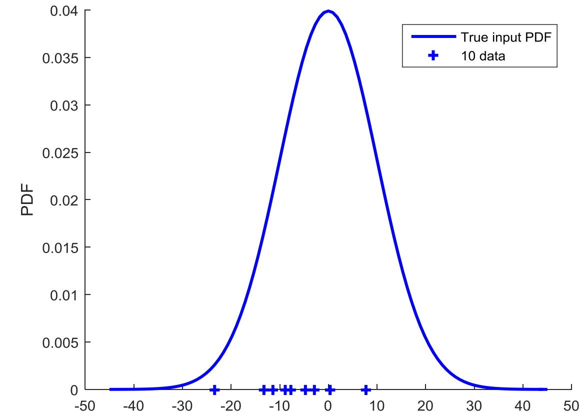
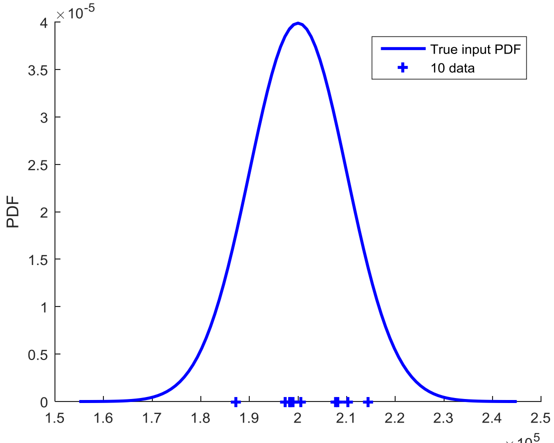


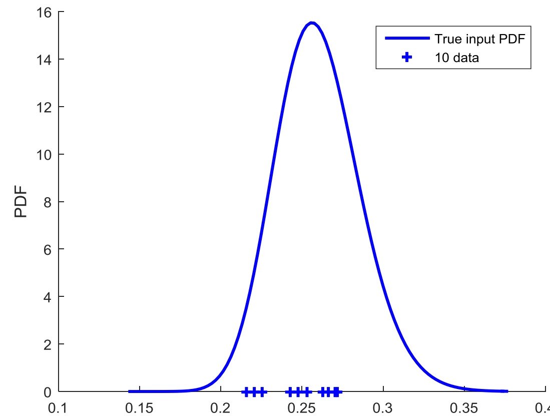
Torsion



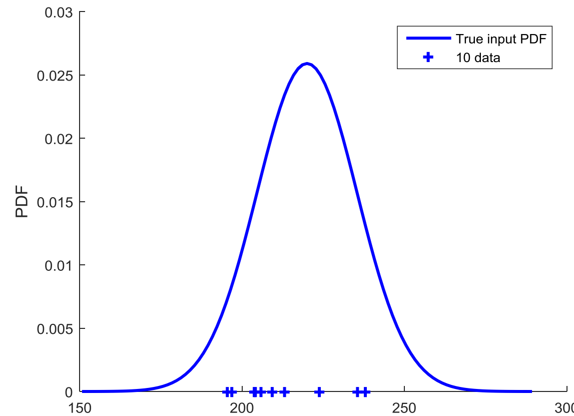
Angle



Young's modulus



Poisson's ratio

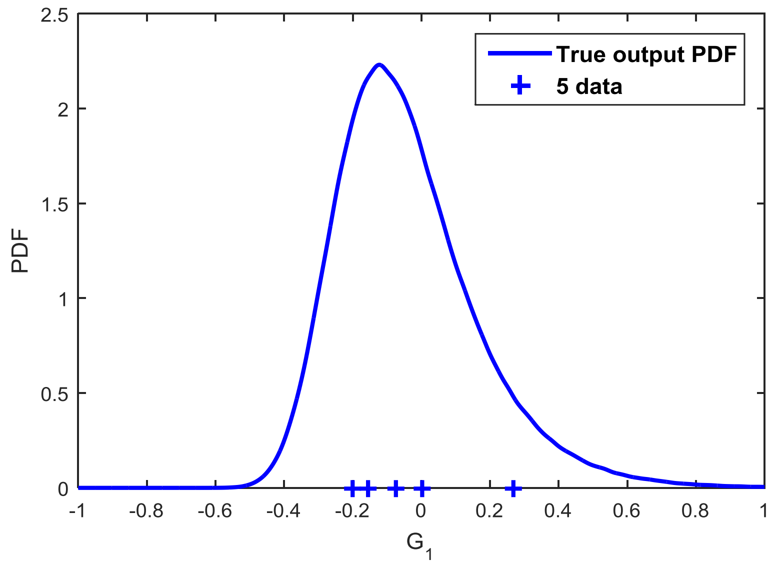


Yield strength

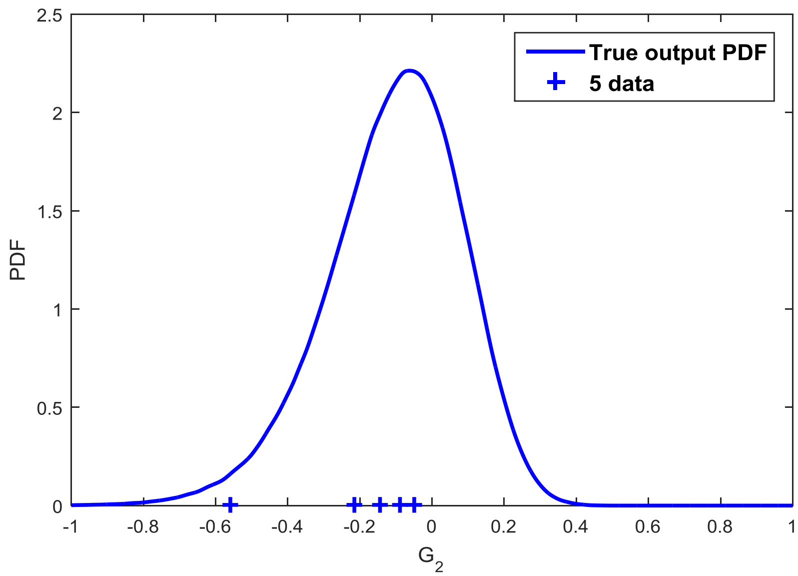
Case 2: Smaller Number of Output Data

- **5 data for each constraint are randomly drawn from true output distributions – to imitate carrying out testing.**

Constraint 1

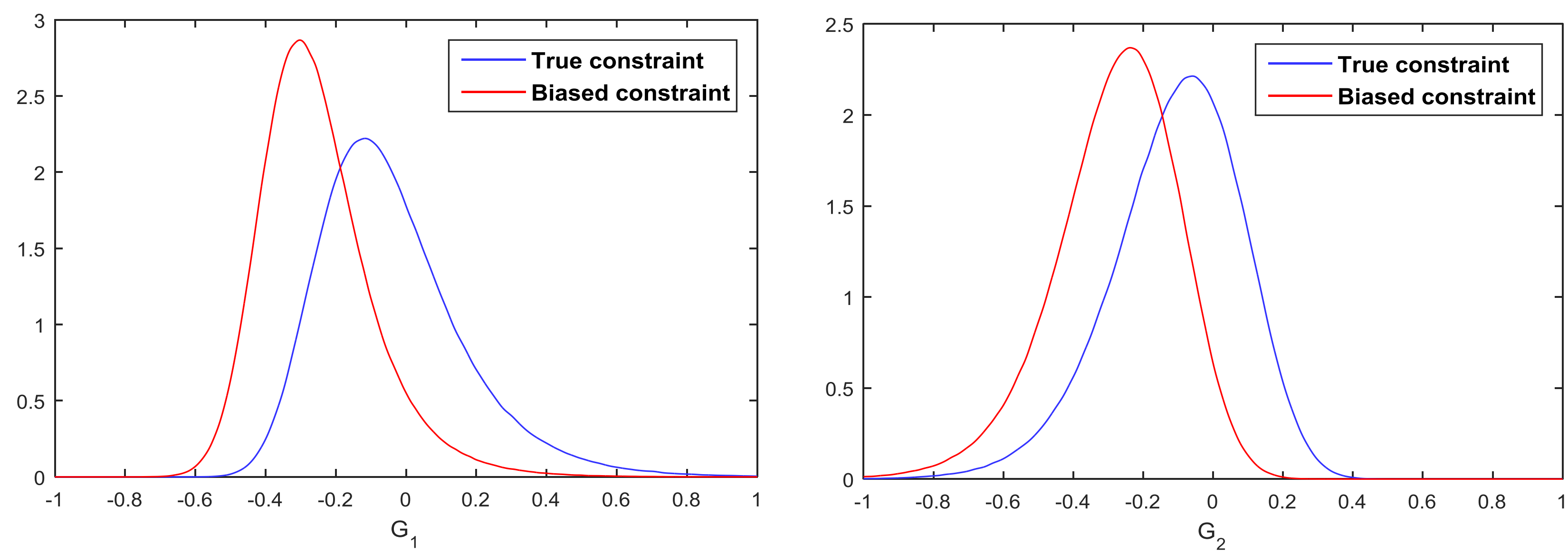


Constraint 2



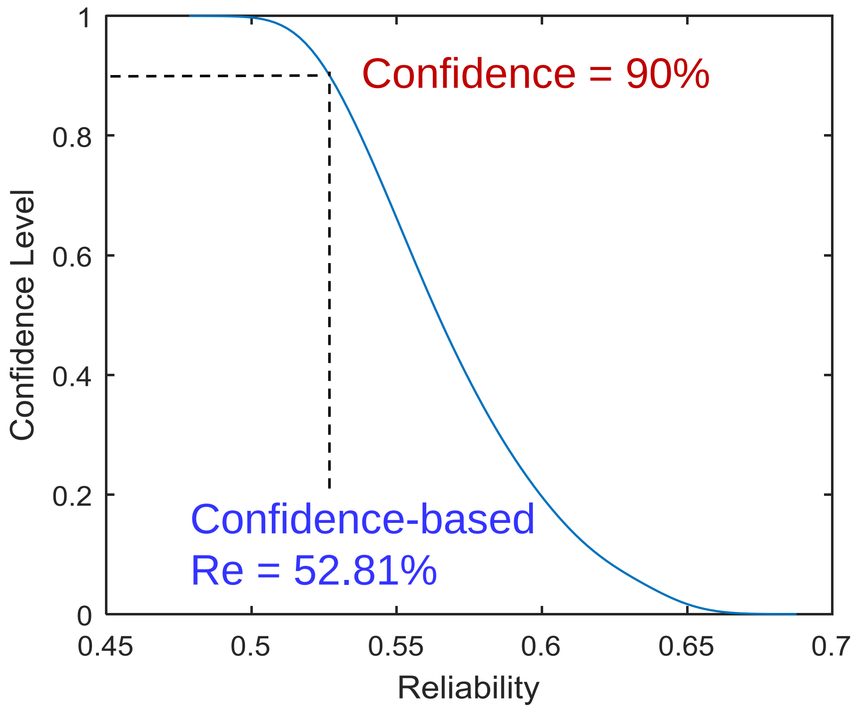
Case 2: Reliability Using Biased Simulation Model

	True constraint (a)		Biased constraint (b)		Error ((a-b)/a×100)	
	$G_1 \le 0$	$G_2 \le 0$	$G_1 \le 0$	$G_2 \le 0$	$G_1 \le 0$	$G_2 \le 0$
Non-normalized output mean	210.14	10,958,273	167.31	12,615,870	20.38%	−15.13%
Reliability using true input dist.	63.91%	69.97%	-	-	-	
Reliability using best-fit input dist.	-	-	93.04%	96.06%	-	

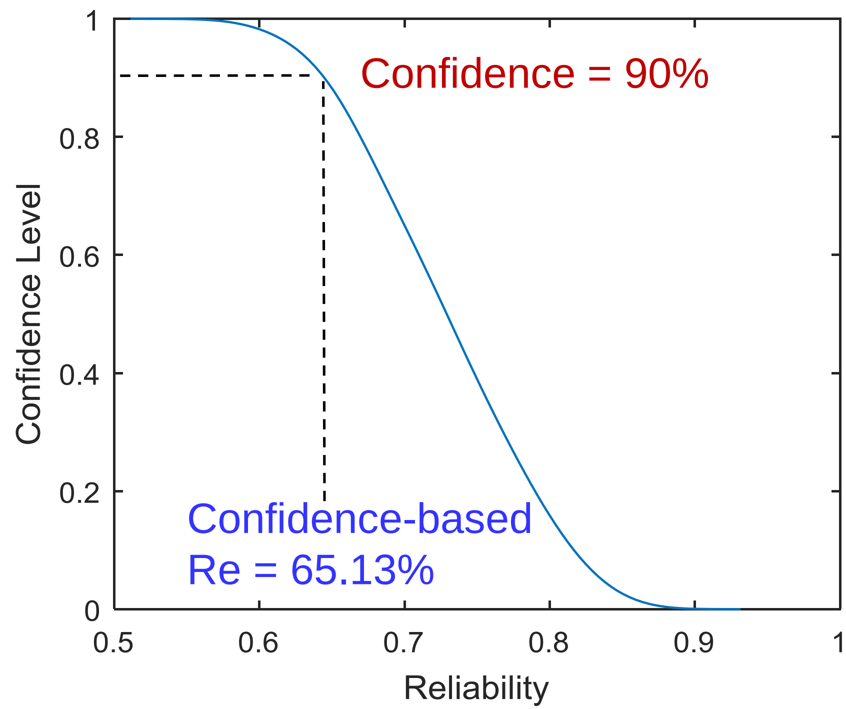


Case 2: Confidence-Based Reliability Assessment

Constraint 1



Constraint 2



RELIABILITY	Constraint 1	Constraint 2
Simulation Only Using Best-fit Input Model	93.04%	96.06%
Best-fit Using Output Test Data Only	67.65%	95.03%
Confidence-based (Simulation + Test Data)	52.81%	65.13%
True	63.91%	69.97%

Conclusions

- Yes we can integrate limited numbers of testing and simulation model to obtain confidence-based UQ and reliability assessment very cost effectively.
- The proposed method should motivate close collaboration between the simulation and testing teams in industry.
- As more test data are used, the proposed method will converge to the true UQ and reliability assessment.
- For the given number of limited data, through 100 trials study, it is found that we obtain better confidence-based reliability results when the data are unbiased and better representation of the true output distribution.
- The proposed method prevents overestimation of reliability even using small number of data.

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Questions?

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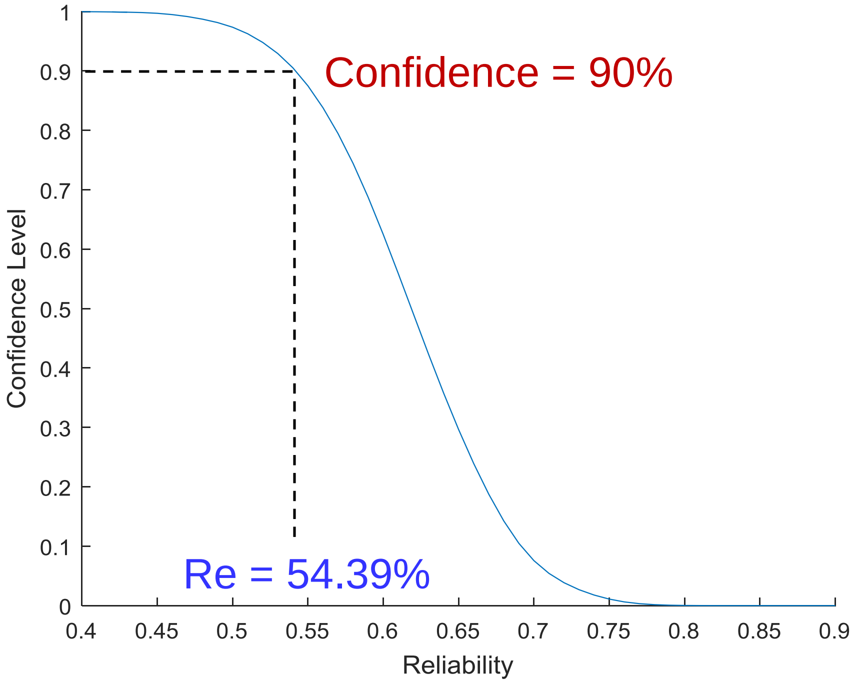
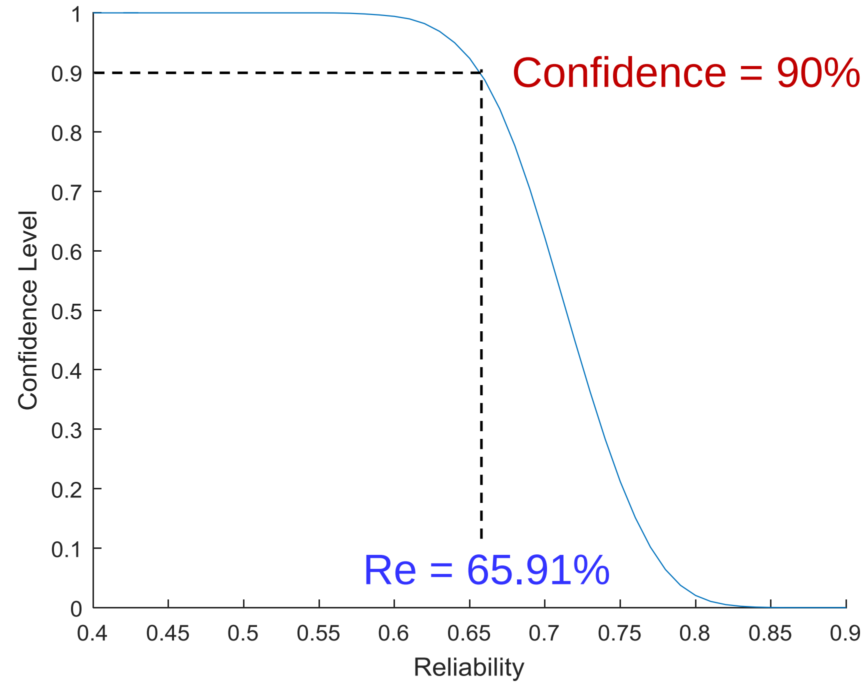
Case 1: Input Uncertainty Only (with True Constraints)

❑ Confidence-based reliability

Constraint 1



Constraint 2



RELIABILITY	Constraint 1	Constraint 2
Confidence-based	54.39%	65.91%
True	63.91%	69.97%

Case 1: Input Uncertainty Only (with True Constraints)

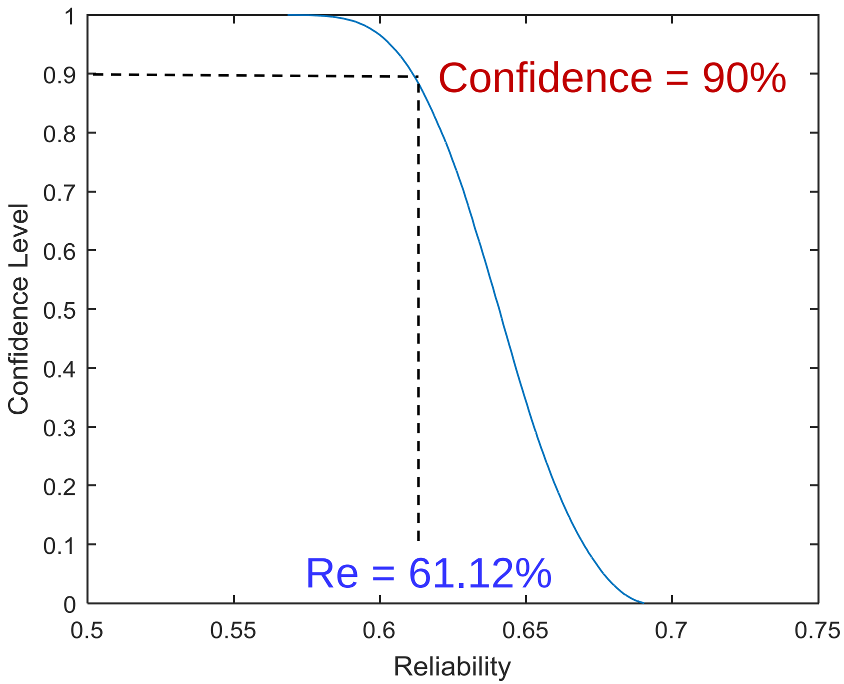
- **Repeated Test: 100 trials for numerical validation**
  - For each trial, 30 data is randomly drawn from true input distribution
- **Target confidence levels of 90% are satisfied**
- **Best-fit method provides confidence level only around 50%**

Method	**Confidence-based**		**Best-fit method**	
Constraint	**1**	**2**	**1**	**2**
Mean	55.07%	64.03%	63.34%	69.87%
STD	6.49%	4.55%	6.26%	4.33%
Maximum	70.82%	73.85%	77.72%	79.15%
Minimum	39.86%	50.06%	49.15%	55.26%
True reliability	63.91%	69.97%	63.91%	69.97%
Successful trials	90	91	51	48

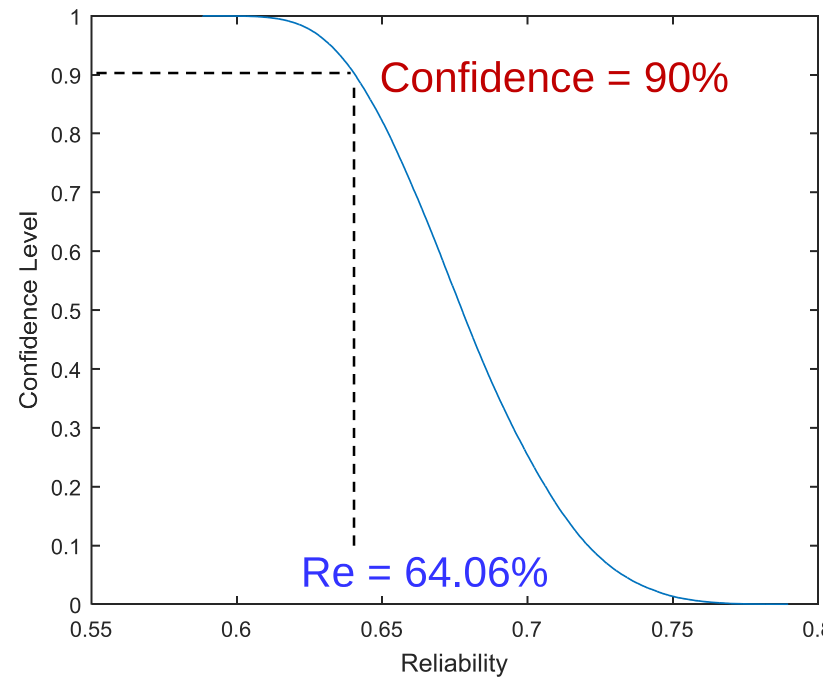
Case 2: Output Uncertainty Only (with True Input)

- **Confidence-based reliability**

Constraint 1



Constraint 2



RELIABILITY	Constraint 1	Constraint 2
Confidence-based	61.12%	64.06%
Best-fit	67.36%	76.39%
True	63.91%	69.97%

Case 2: Output Uncertainty Only (with True Input)

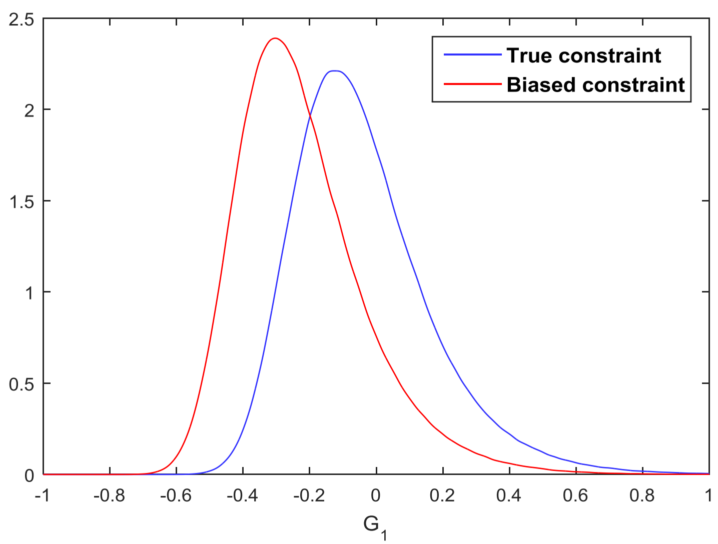
- **Repeated Test: 100 trials for numerical validation**
  - For each trial, 20 data is randomly drawn from true output distribution
- **Target confidence levels of 90% are satisfied**
- **Best-fit method provides confidence level only around 65%**

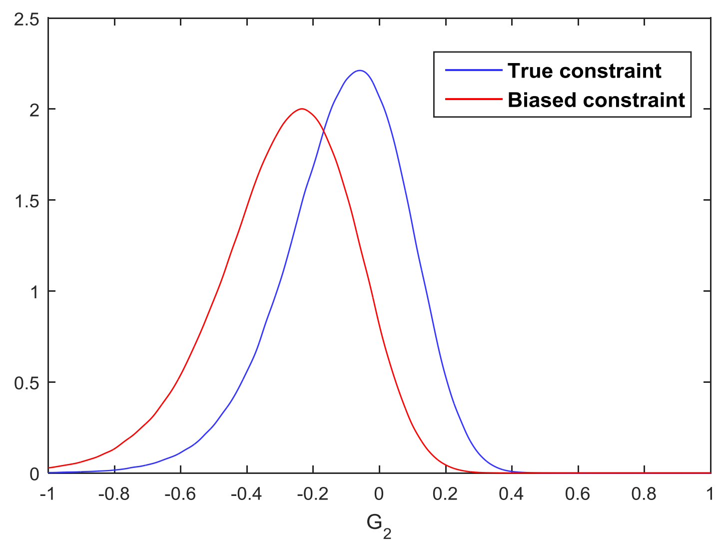
Method	**Confidence-based**		**Best-fit method**	
Constraint	**1**	**2**	**1**	**2**
Mean	57.35%	62.65%	61.59%	67.34%
STD	5.89%	5.17%	8.46%	7.71%
Maximum	70.46%	78.37%	84.31%	92.01%
Minimum	45.74%	50.63%	44.37%	51.25%
True reliability	63.91%	69.97%	63.91%	69.97%
Successful trials	89	92	65	67

Example Description (2-sigma design point)

❑ Biased Simulation Model

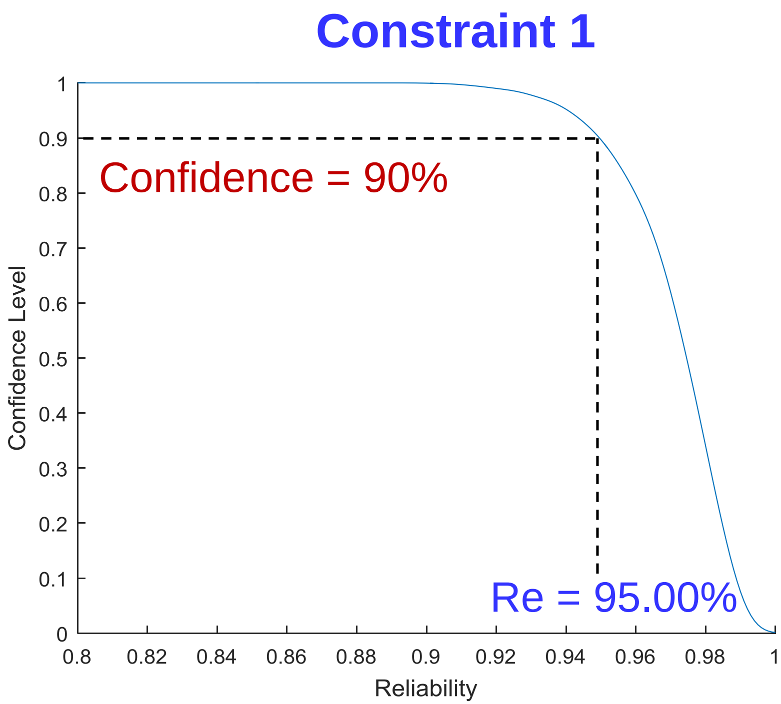
	True constraint (b)		Biased constraint (a)		Error ((b-a)/b×100)	
	$G_1 \le 0.4735$	$G_2 \le 0.2203$	$G_1 \le 0.4735$	$G_2 \le 0.2203$	$G_1 \le 0.4735$	$G_2 \le 0.2203$
Non-normalized output mean	210.14	10,958,273	167.31	12,615,870	**20.38%**	**−15.13%**
Reliability using true input dist.	**97.73%**	**97.73%**	-	-	-	
Reliability using best-fit input dist.	-	-	**99.62%**	**99.90%**	-	

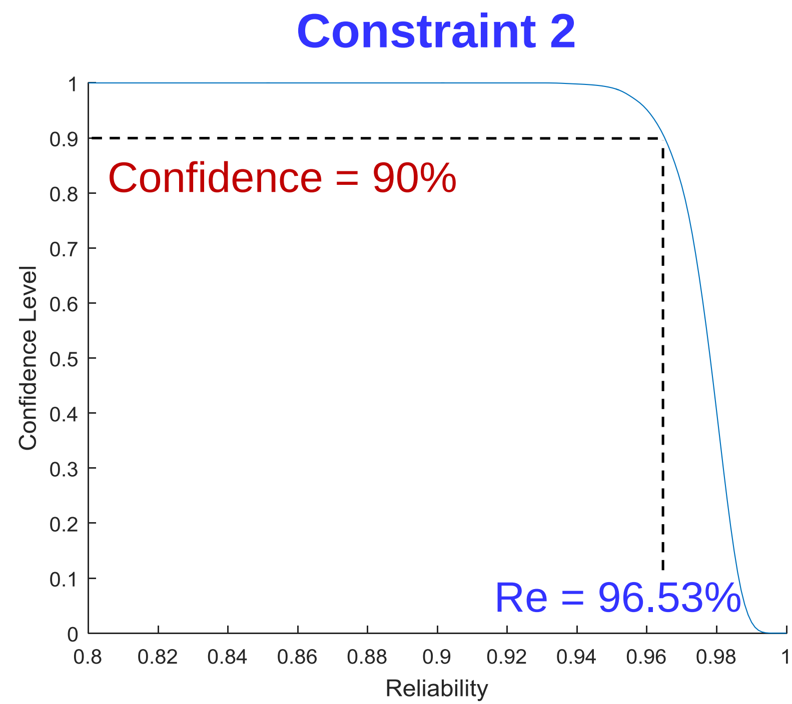




Case 1: Input Uncertainty Only (with True Constraints)

- **Confidence-based reliability at 2-sigma design point**



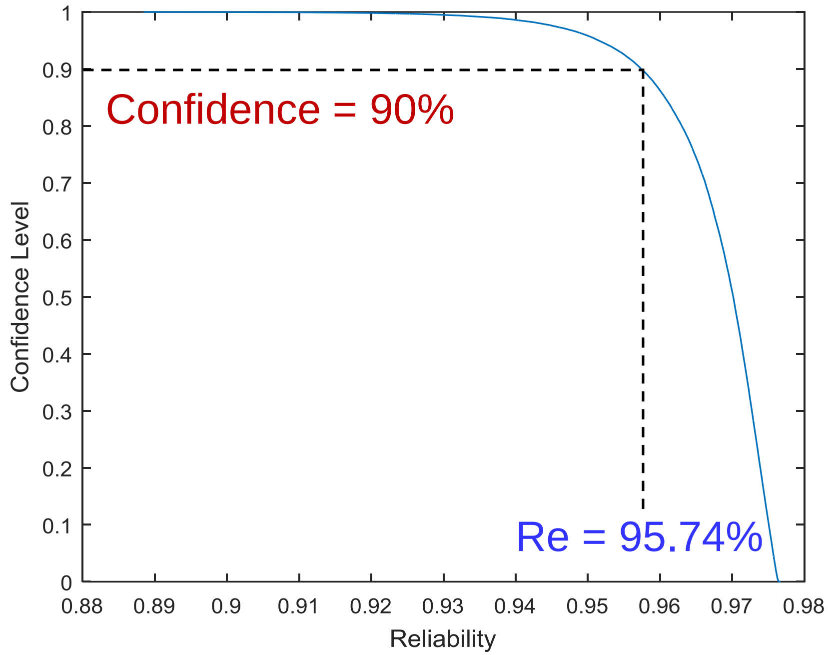


RELIABILITY	Constraint 1	Constraint 2
Confidence-based	95.00%	96.53%
True	97.73%	97.73%

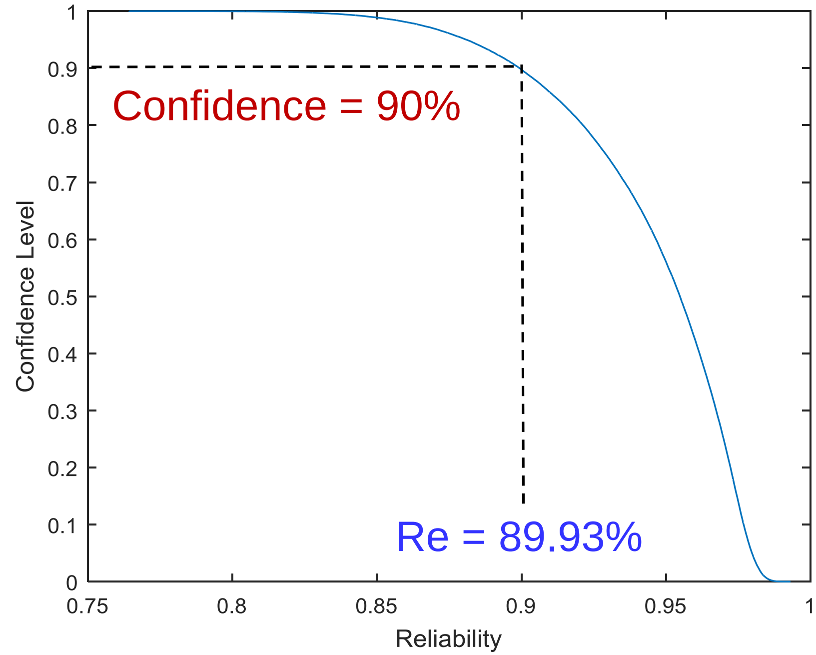
Case 2: Output Uncertainty Only (with True Input)

- Confidence-based reliability at **2-sigma design point**

Constraint 1



Constraint 2

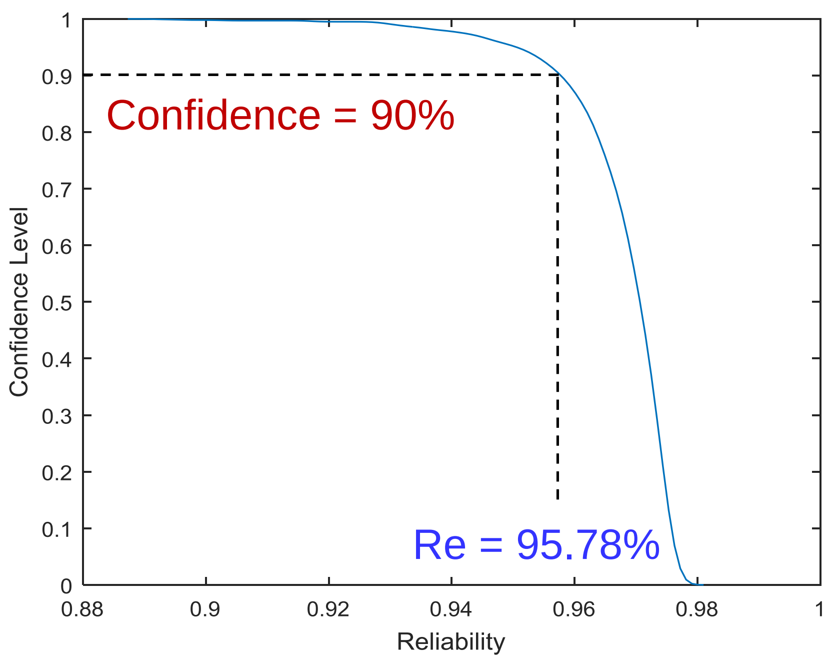


RELIABILITY	Constraint 1	Constraint 2
Confidence-based	95.74%	89.93%
True	97.73%	97.73%

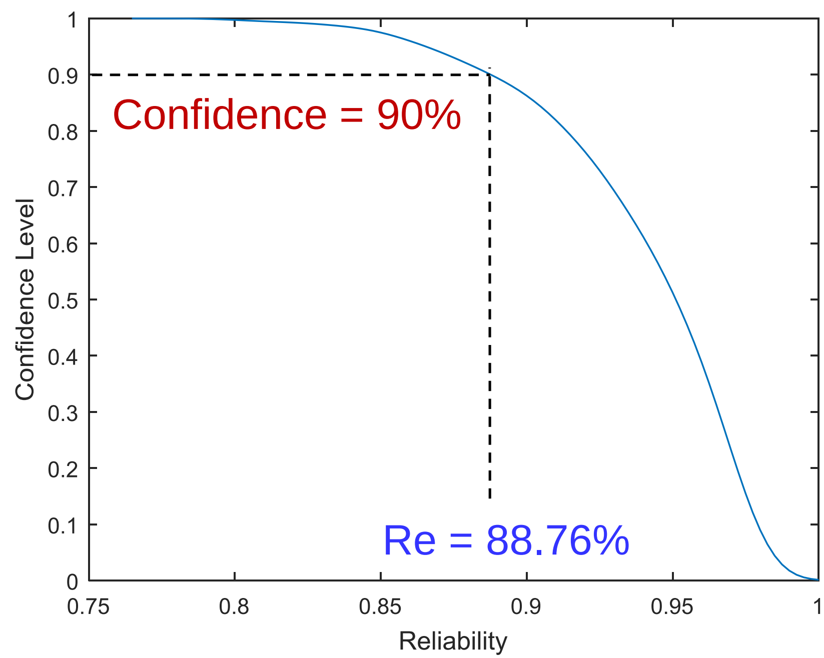
Case 3: Both Input and Output Uncertainties

- Confidence-based reliability at **2-sigma design point**

Constraint 1



Constraint 2



RELIABILITY	Constraint 1	Constraint 2
Confidence-based	95.78%	88.76%
True	97.73%	97.73%